ON THE TRANSITION FROM HOMOGENEOUS TO BUBBLING FLUIDIZATION

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The onset of the aggregative mode of liquid–solid fluidization was explored. The experimental findings were interpreted by means of the dynamic (elastic) wave velocity and the voidage propagation (continuity) wave velocity. For widely different systems, the mapping of regimes has been presented in terms of the Archimedes number, the Froude number and the fluid–solid density ratio. The proposed diagram also depicts the typical Geldart's Group A particles fluidized with air.
Key words: Particulate and aggregative fluidization; Transition; Mapping of regimes.

A liquid can be employed to fluidize a mass of solid particles[1]. Similarly as in gas fluidization, the system is characterized by a minimum fluidization velocity, U_{mf}, at which the drag force exerted on the particles by the upward flowing fluid just balances the effective particle weight (weight minus buoyancy force)[2,3]. For velocities smaller than U_{mf}, the bed is in the fixed (static) state condition, for velocities greater than U_{mf}, the bed is fluidized. It expands either homogeneously (a particulate, smooth, non-bubbling manner of fluidization) as the fluid velocity is increased or the amount of fluid in excess of U_{mf} bypasses the bed as bubbles (a heterogeneous, aggregative, bubbling mode of fluidization). By the bubbles we mean different-shaped pockets (inhomogeneities) of fluidizing fluid almost free of particles. Unlike the gas–solid systems, the majority of liquid fluidized beds tend to fluidize without forming bubbles.

Current industrial applications of gas fluidized beds are much more numerous than those of liquid fluidized beds. Quite recently, however, new processes are being explored involving the use of liquid–solids beds in the areas of water treatment, hydrometallurgy, food production and biochemical processing[4].

The purpose of this article is to explore the transition between particulate and aggregative fluidization, *i.e.*, the minimum bubbling conditions for liquid fluidized beds.

THEORETICAL

Homogeneous (Particulate) Expansion

It is commonly accepted that liquid fluidized beds expand in a homogeneous mode and a flat, sharp interface between the top of bed and freeboard is generally preserved. One should realize that the effects of factors on the bed expansion such as particle irregularities, particle size distributions should always be taken into account. Moreover, the influence of the distributor design and the ratio of particle size to column diameter can also be significant. This work is confined to spherical, nearly monosized particles fluidized in vessels at which the effects of the physical boundaries are negligible.

The increase of bed voidage with the increasing superficial velocity of fluid, U, is described by a purely empirical relationship (*1*) which is generally known as the Richardson–Zaki equation[1]

$$U/U_t = Re/Re_t = \varepsilon^n \ , \qquad (1)$$

where the exponent, n, is a function of the Reynolds number under terminal free-fall conditions, Re_t. For different ways of predicting Re_t, a reader is referred to a recent article of ours[5]. The values of Re_t were also rigorously computed for $Ar \in \langle 1, 4 \,.\, 10^7 \rangle$ and tabulated in this work[5]. Garside and Al-Dibouni[6] proposed that n should be evaluated from

$$n = \frac{5.09 + 0.284\, Re_t^{0.877}}{1.0 + 0.104\, Re_t^{0.877}} \qquad (2)$$

for $Re_t \in \langle 10^{-3}, 3 \,.\, 10^4 \rangle$.

In order to avoid the prior prediction of Re_t in the estimation of n, Khan and Richardson[7] expressed the exponent n with the aid of the Archimedes number as

$$n = \frac{4.8 + 0.103\, Ar^{0.57}}{1.0 + 0.043\, Ar^{0.57}} \qquad (3)$$

for $Ar \in \langle 0.1, 10^8 \rangle$.

Comparison of the empirical correlations (*2*) and (*3*) for the Richardson–Zaki exponent is presented in Fig. 1. The differences between the values of n predicted by the newer correlations (*2*) and (*3*) are about 10%. The relation between Ar and Re_t is given below by Eqs (*12*) and (*13*). A sharp drop in the exponent, n, when moving from the viscous flow regime ($Re_t < 1$) to the Newtonian flow regime ($Re_t > 1\,000$) is visualized

in Fig. 1. Similar correlations of other authors for the exponent n are reviewed and compared elsewhere[4,8].

Other relationships have been developed which also describe, more or less, satisfactorily, the log-linear expansion characteristics expressed by Eq. (*1*). Utilizing the data of Garside and Al-Dibouni[6], we obtained a common voidage function for the entire flow regime[9]

$$1.440Re^{1.804} + 20.36Re - \varepsilon^{4.730}Ar = 0 \qquad (4)$$

for $Re \in \langle Re_{mf}, Re_t \rangle$ and $\varepsilon \in \langle \varepsilon_{mf}, 1 \rangle$.

It is apparent that, in addition to accuracy and convenience, a desired correlation should also describe the boundary situations, *i.e.*, the point of minimum fluidization (Re_{mf}, ε_{mf}) and the free-fall conditions (Re_t, $\varepsilon = 1$). With respect to a very wide range of pertinent flow conditions ($Re_t \in \langle 10^{-3}, 10^4 \rangle$), these requirements are very severe.

Onset of Aggregative (Bubbling) Behaviour

The occurrence of bubbling phenomena in liquid fluidized beds was first reported almost fifty years ago by Wilhelm and Kwauk[10] in work with lead shots and water. The behaviour of bubbling liquid and gas fluidized beds is similar in many aspects. For example, voids in liquid fluidized beds correspond to the passage of gas bubbles in beds fluidized with gas. The start of aggregative behaviour can be easily detected in both the systems for example by measuring the pressure fluctuations within the bed.

Wallis[11] considered the occurrence of particle-phase pressure which leads to the existence of dynamic waves in solid–fluid systems. Such waves may either grow into shocks (voids, bubbles) or decay depending on the magnitude of their velocity relative

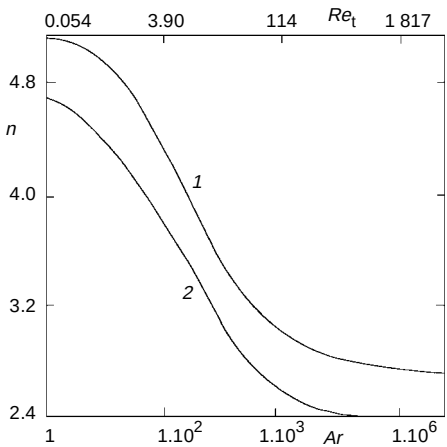

FIG. 1
Comparison of the correlations for the Richardson–Zaki exponent n: *1* according to Garside and Al-Dibouni[6], Eq. (*2*); *2* Khan and Richardson[7], Eq. (*3*)

to that of the continuity waves. This point, *i.e.*, the development of bubbles (voids) from the growth of small perturbations, is still a matter of interest and some controversy. According to the studies of Wallis[11,12], and Wallis *et al.*[13] the system is stable (particulate behaviour) if the dynamic (elastic) wave velocity, U_e, is greater than the continuity wave velocity, U_ε. The system is unstable (aggregative, bubbling behaviour) if the continuity wave velocity exceeds the dynamic wave velocity.

Foscolo and Gibilaro[14–16] developed a dynamic, fluid-particle interaction model of a particulate fluidized bed. Their model made it possible to formulate the general stability criterion of Wallis[11] for the onset of aggregative behaviour quite specifically. Based on this model, the elastic wave velocity (that is analogous to the velocity of sound in a compressible fluid) is given as

$$U_e = [3.2gd_p(1 - \varepsilon)(\rho_s - \rho_f)/\rho_s]^{1/2} \quad . \tag{5}$$

The quantity U_e expresses the maximum velocity at which the small voidage disturbances can propagate throughout a bed without developing shock fronts (inhomogeneities, voids, bubbles).

The voidage propagation velocity, U_ε, is expressed by

$$U_\varepsilon = nU_t(1 - \varepsilon)\varepsilon^{n-1} \quad . \tag{6}$$

The Wallis's stability criterion is stated as follows:

$$U_e \begin{matrix} > \\ = \\ < \end{matrix} U_\varepsilon \ , \tag{7}$$

where the sign > holds for stable (particulate, homogeneous) behaviour, the sign < holds for unstable (aggregative, heterogeneous) behaviour and the sign = holds for the stability limit (the boundary line between homogeneous and heterogeneous behaviour). It can be simply expressed with the use of Eqs (*5*) and (*6*).

When $U_e > U_\varepsilon$, the voidage disturbances decay rapidly to the equilibrium condition and the system is stable (homogeneously expanded bed). On the other hand, when $U_e < U_\varepsilon$ the disturbances tend to grow rapidly, inhomogeneities develop, and the systems is unstable (heterogeneously expanded bed). In the operation regime for which $U_e = U_\varepsilon$, a stability limit is just attained. In other words, the equality of U_e and U_ε defines the boundary between homogeneous (particulate) and heterogeneous (aggregative) behaviour of a fluidized bed. The equality $U_e = U_\varepsilon$ determines the voidage at which a possible

transition between particulate and aggregative fluidization occurs. On this basis, the point of minimum bubbling, explored for example by Broadhurst and Becker[17], can be defined as the condition where the dynamic and continuity wave velocities become equal.

Although the above relationships may seem formally simple, the general hydrodynamic behaviour of the fluid–solid systems is an intricate one. This is demonstrated, for example, by the dependence of the exponent, n, on Re_t or Ar shown in Fig. 1. Moreover, the velocity of the continuity wave, U_ε, given by Eq. (*6*), is a non-linear function of the voidage, ε, and exhibits an extremal behaviour.

The critical condition at which the bubbling just starts, *i.e.*, the minimum bubbling criterion, can also be expressed in terms of the common dimensionless groups (Ar, Re_t and De). On introducing these non-dimensional groups into Eqs (*5*)–(*7*), we can get after some manipulations for the onset of bubbling (boundary between homogeneous and heterogeneous behaviour, stability limit)

$$(Ar\,De)^{1/2} - \left(\frac{1-\varepsilon}{3.2}\right)^{1/2} n\varepsilon^{n-1} Re_t = 0 \quad [U_e = U_\varepsilon] \tag{8}$$

or

$$Fr/(1-De) - 3.2/[(1-\varepsilon)n^2\varepsilon^{2n-2}] = 0 \quad [U_e = U_\varepsilon] \tag{9}$$

where De is the density ratio, $De = \rho_f/\rho_s$, and the Froude number, Fr, is defined as

$$Fr = U_t^2/(gd_p) = (Re_t^2/Ar)\,(1-De)/De \tag{10}$$

or

$$Fr = Re_t^2/Ga \ . \tag{11a}$$

In other words, the original Wallis's criterion, given by Eq. (*7*), for stable, particulate behaviour (homogeneous expansion) can be expressed with respect to the above lines as

$$Fr/(1-De) < 3.2/[(1-\varepsilon)n^2\varepsilon^{2n-2}] \quad [U_e > U_\varepsilon] \ . \tag{11b}$$

EXPERIMENTAL

All experiments were performed at 25 °C in a glass fluidization column of internal diameter D = 51 mm. The column was provided with a perforated plate distributor with 0.5 mm circular orifices. The relative area of the perforations was as large as 0.63%. With the use of a thermostat, the temperature in the bed section was maintained within 0.2 K along its length. A pump was employed to circulate the fluidizing liquid (water), and the flow rate was measured by a calibrated rotameter manifold.

The distilled water was employed as the fluidizing fluid. Its literature values of density and dynamic viscosity amount to ρ_f = 996.9 kg m^{-3} and μ_f = 0.8937 mPa s, respectively.

Three fractions (sieve cuts) of steel spheres ($\bar{d}_p$ = 0.2, 0.5 and 1.0 mm) and two fractions of glass spheres ($\bar{d}_p$ = 1.5 and 2.5 mm) were investigated in this study. The solid densities were found to be as large as ρ_s = 7 700 kg m^{-3} (steel) and ρ_s = 2 700 kg m^{-3} (glass). The basic physical parameters of the above systems are given in Table I.

The voidage of bed was determined from the measured height of expanded bed, H, according to relationship

$$\varepsilon = 1 - W/(FH\rho_s) \quad . \tag{11c}$$

RESULTS AND DISCUSSION

The experiments were performed to observe the bed homogeneity (fluidization quality) for the range of voidage $\varepsilon \in \langle 0.4, 0.9 \rangle$. The flow rate of water was increased gradually from the onset of fluidization ($U = U_{mf}$, $\varepsilon = \varepsilon_{mf} = 0.4$) until the observations of bed surface became difficult due to the turbulent vortexing at high fluid velocities.

It is apparent that the beds of steel particles are apt to expand aggregatively more than glass particles because of their markedly higher density (ρ = 7 700 *vs* 2 700 kg m^{-3}). Three fractions of steel spheres were used to examine expansion of such fluidized suspensions. Smooth fluidization of the smallest particles ($\bar{d}_p$ = 0.2 mm) was observed. The

TABLE I
Physical properties of the spherical solids and terminal velocity conditions in water at 25 °C

Material	Size range mm	$\bar{d}_p$ mm	ρ_s kg m^{-3}	*Ar*	Re_t	U_t m s^{-1}	*n* Eq. (*3*)	*Fr*
Steel	0.15–0.25	0.2	7 700	656.5	17.2	0.0771	3.77	3.03
	0.55–0.45	0.5		10 260	115.2	0.206	2.66	8.70
	0.9–1.1	1.0		82 060	429.5	0.385	2.49	15.1
Glass	1.4–1.6	1.5	2 700	70 370	391.0	0.234	2.49	3.71
	2.3–2.7	2.5		325 680	972.0	0.348	2.44	4.95

Properties of water at 25 °C: ρ_f = 996.9 kg m^{-3}, μ_f = 0.8937 mPa s.

bed expanded in a uniform manner with increasing fluid velocity and the transition to aggregative behaviour did not occur. However, this was not the case of the larger particles ($\bar{d}_p$ = 0.5 mm) in which voidage pockets become visible at a voidage of about 0.5. At this moment, the bed surface, originally flat and well-defined, showed fluctuations that considerably increased with further increasing fluid velocity. Such an appearance is characteristic of beds fluidized with gas. The behaviour of the largest steel particles (d_p = 1.0 mm) was not quite easy to classify in the vicinity of the point of minimum fluidization. Smaller voidage discontinuities were likely to occur at the column walls. As the liquid velocity was somewhat increased, pockets of liquid were clearly visible and considerable fluctuations of the bed surface occurred.

In the experiments with the glass spheres ($\bar{d}_p$ = 1.5 and 2.5 mm), fully particulate expansion was not detected. Within the bed of the 1.5 mm particles, quite narrow liquid pockets appeared at a bed voidage of around 0.6. The voidage discontinuities spanned the entire column cross section and tended to break as the liquid superficial velocity was increased. Flat voids appeared in the bed of the larger particles ($\bar{d}_p$ = 2.5 mm) at $\varepsilon \div 0.4$–0.5. The visual observations of both systems indicate that the transition from particulate to aggregative behaviour is more distinct for smaller particles than for larger ones. More or less diffusive transition appears to be characteristic of large suspended particles.

Comparison with Theory

As follows from Eqs (*1*)–(*4*), the voidage of liquid–solid fluidized beds is an intricate function of the flow conditions, *Re*, and the Archimedes number, *Ar*. As we discussed recently in ref.[8] and as illustrated in Fig. 1, there are appreciable differences in predicting the velocity–voidage relationships. On the other hand, the free fall conditions of isolated spheres at steady-state given by

$$C_D Re_t^2 = \frac{4}{3} Ar \qquad (12)$$

can be estimated quite accurately with the aid of the Turton–Levenspiel correlation for the drag coefficient[18]

$$C_D = \frac{24}{Re_t}(1 + 0.173\, Re_t^{0.657}) + \frac{0.413}{1 + 16\,300\, Re_t^{-1.09}} \ . \qquad (13)$$

Predictions for nonspherical particles can be, however, substantially less accurate[19].

The Wallis's stability criterion may simply be expressed in dimensionless form through the function F_U introduced as

$$F_U = (U_e - U_\varepsilon)/U_e \quad . \tag{14}$$

The quantity F_U can readily be evaluated as a function of bed voidage, ε. Positive and negative values of F_U reflect homogeneous and heterogeneous behaviour, respectively. Zero value of F_U pinpoints a voidage at which the transition from homogeneous to heterogeneous fluidization is predicted to occur. Using Eqs (*5*), (*6*) and (*14*), we have made systematic predictions of the function F_U for different ε. Dimensionless criteria of stability alternative to Eq. (*14*) may be expressed on the basis of Eq. (*8*) or (*9*).

As can be seen above, the purely empirical Richardson–Zaki exponent[1], *n*, is an important quantity. This exponent was predicted with the aid of Eq. (*3*).

Values of the criterion (*14*) were systematically computed for the systems explored by experiment and plotted in Figs 2 and 3 (as a function of bed voidage). As demonstrated by curves *1* and *2* in Fig. 2, the smallest steel particles ($\bar{d}_p$ = 0.2 mm) expand fully in the particulate manner ($F_U > 0$). Equation (*6*) suggests and curves *1* and *2* in Fig. 2 visualize that greater values of the Richardson–Zaki exponent *n* (as predicted by Eq. (*2*)) signal the lower stability of moderately expanded beds with the same particles. Particles with $\bar{d}_p$ = 0.5 mm pass through

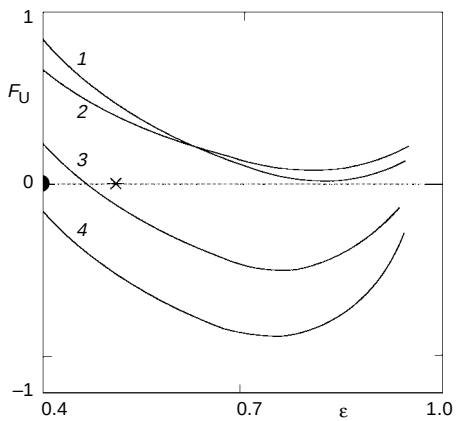

FIG. 2

Mode of fluidization of steel spheres by ambient water indicated by the stability criterion (*14*) as a function of bed voidage and particle diameter. *1* $\bar{d}_p$ = 0.2 mm, *n* = 3.28 (predicted by Eq. (*3*)); *2* $\bar{d}_p$ = 0.2 mm, *n* = 3.77 (predicted by Eq. (*2*)); *3* $\bar{d}_p$ = 0.5 mm, *n* = 2.7 (predicted by Eq. (*3*)), * experimental point of transition; *4* $\bar{d}_p$ = 1.0 mm, *n* = 2.5 (predicted by Eq. (*3*)), ● experimental point of transition

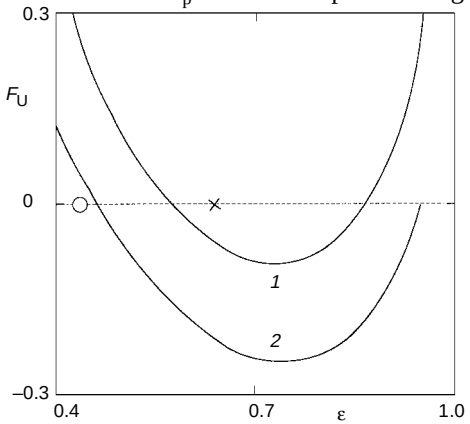

FIG. 3

Mode of fluidization of glass spheres by ambient water indicated by the stability criterion (*14*) as a function of bed voidage and particle diameter. *1* $\bar{d}_p$ = 1.5 mm, *n* = 2.5 (predicted by Eq. (*3*)), × experimental point of transition; *2* $\bar{d}_p$ = 2.5 mm, *n* = 2.4 (predicted by Eq. (*3*)), ❍ experimental point of transition

both the fluidization regimes and particles with $\bar{d}_p$ = 1.0 mm expand always aggregatively from the onset of fluidization. These model estimates are in reasonable agreement with the experimental findings presented in Figs 2 and 3.

The second intersection of F_U with the abscissa in Fig. 3 marks a return from aggregative to particulate behaviour at a higher bed voidage. Unfortunately, this interesting point was difficult to examine because of fluctuations of the bed surface and a given capacity of the circulation pump.

Similarly, values of the Wallis's criterion were also evaluated for the glass particles ($\bar{d}_p$ = 1.5 and 2.5 mm). The computed results are plotted as a function of the bed voidage in Fig. 3. As can be seen, the mode of fluidization passes through both regimes of expansion. This is also in qualitative agreement with the experimental observations.

Several decades ago, Wilhelm and Kwauk[10] proposed the Froude number defined by Eq. (*10*) as a practical criterion for discriminating between the particulate and aggregative behaviour of fluidized beds. These authors employed this parameter purely on an empirical basis without physical reasoning. They reported that the fluidized bed behaved aggregatively for $Fr > 1$.

With respect to Eqs (*8*) and (*9*), it is apparent that rather than the Froude number alone either of the equivalent groups

$$Fr/(1 - De) = Re_t^2/(Ar\ De) \qquad (15)$$

should rather be employed for predicting the point of transition from the homogeneous to the bubbling fluidization.

The above criterion is employed below in efforts to draw a global map of particulate and aggregative behaviour of beds fluidized with different fluids.

The point at which the curve $F_U = F_U(Ar, \varepsilon)$ just touches the horizontal axis corresponds to the critical (maximum) value of $Fr/(1 - De)$ for which the system still behaves particulately throughout the whole range of expansion. Such critical values of $Fr/(1 - De)$ were systematically estimated from Eq. (*9*) for $Ar \in \langle 1, 10^7 \rangle$ and $\varepsilon = 0.8$ with the aid of the empirical correlation (*3*) for the Richardson–Zaki exponent *n*. The computed results are shown in Figs 4 (line *1*) and 5.

The other critical value of the criterion $Fr/(1 - De)$ corresponds to a situation where the curve F_U intersects the abscissa already at the voidage equal to the bed voidage at the point of minimum fluidization, *i.e.*, at $\varepsilon = \varepsilon_{mf} = 0.4$. Such states are delineated by curve *2* in Fig. 4. This curve marks the boundary of the systems which expand aggregatively from the onset of fluidization.

The area between lines *1* and *2* in Fig. 4 represents the systems that expand particulately or aggregatively in dependence on the corresponding voidage of bed and thus on

the actual fluid velocity. As demonstrated in this figure, the experimental data points – though not numerous – support the model predictions.

The region *3* in Fig. 4 represents typical Group A particles in the Geldart's diagram[20,21] for classifying materials that are very easy to fluidize with gas. Such powders are characterized by a relatively small particle size (typically materials with $\overline{d}_p$ = 50–100 μm) and a low particle density (typically ρ_s = 600–1 100 kg m^{-3}). Fluid cracking catalysts are typical examples of these fine and light powders. Their gas fluidized beds undergo considerable expansion at gas velocities between the minimum fluidization velocity, U_{mf}, and the velocity at which bubbling commences (the minimum bubbling point, U_{mb}). The minimum bubbling velocity, U_{mb}, is always greater than the minimum fluidization velocity, U_{mf}. In other words, the beds of Group A solids exhibit both aggregative and particulate behaviour, which is in agreement with the theoretical predictions shown in Fig. 4.

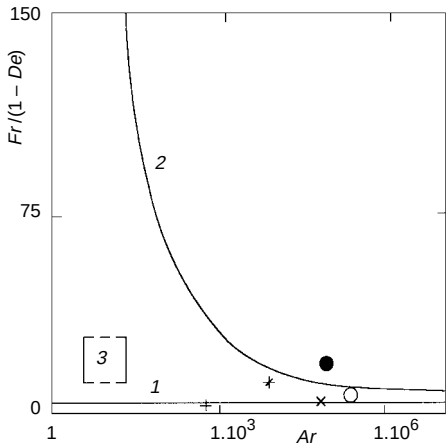

FIG. 4

Mapping of the mode of fluidization: *1* predicted boundary below which fully particulate expansion occurs, ε = 0.8; *2* predicted boundary above which fully aggregative expansion occurs, ε = 0.4; *3* empirical region representing typical, very-easy-to–fluidize, Group A solids of the Geldart classification[20] which exhibit particulate-aggregative behaviour when fluidized by ambient air (*Ar* ≈ 10, *Fr* ≈ 15); experimental data (fluidization with ambient water): + ρ_s = 7 700 kg m^{-3}, $\overline{d}_p$ = 0.2 mm; * ρ_s = 7 700 kg m^{-3}, $\overline{d}_p$ = 0.5 mm, ● ρ_s = 7 700 kg m^{-3}, $\overline{d}_p$ = 1.0 mm; × ρ_s = 2 700 kg m^{-3}, $\overline{d}_p$ = 1.5 mm; ❍ ρ_s = 2 700 kg m^{-3}, $\overline{d}_p$ = 2.5 mm

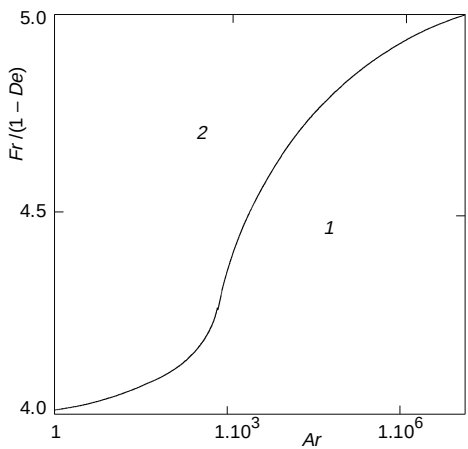

FIG. 5

Mapping of the mode of fluidization. The solid line shows the predicted bounds of fully particulate expansion (enlarged section of Fig. 4, with line *1*); *1* fully particulate fluidization; *2* particulate and aggregative fluidization

Figure 5 indicates that the bounds of particulate fluidization intuitively assumed by Wilhelm and Kwauk[10] ($Fr \approx 1$ for $Ar \approx 5$ and $De << 1$) seem to be somewhat conservative. Nevertheless, such values are not far from the predictions shown in Fig. 5.

CONCLUSIONS

The transition from particulate to aggregative mode of fluidization can be conveniently predicted with the aid of the general stability criterion of Wallis[11] and the particle bed model of Foscolo and Gibilaro[14]. The flow conditions at which the onset of heterogeneities (bubbles) occurs (point of minimum bubbling) are an intricate function of the Archimedes criterion, the fluid–solid density ratio and the bed voidage. The commencement of a bubbling regime is more easily observed for smaller particles. The present experience suggests that the proposed mapping of behaviour of liquid fluidized beds is at least in qualitative agreement also with the entirely empirical Geldart classification diagram of solids for their fluidizing with air.

SYMBOLS

Ar	Archimedes number, $Ar = d_p^3 g \rho_f (\rho_s - \rho_f)/\mu_f^2$
C_D	drag coefficient of particle, $C_D = 4gd_p(\rho_s - \rho_f)/(3U_t^2\rho_f)$
d_p	diameter of particles, m
$\underline{d}_p$	mean diameter of particles, m
De	fluid–solid density ratio, $De = \rho_f/\rho_s$
F	cross-sectional area of column, m^2
F_U	stability function defined by Eq. (*14*)
Fr	Froude number, $Fr = U_t^2/(gd_p) = Re_t^2/Ga = (Re_t^2/Ar)(1 - De)/De = (Re_t^2/Ar)(\rho_s - \rho_f)/\rho_f$
g	acceleration due to gravity, $g = 9.807$ m s^{-2}
Ga	Galileo number, $Ga = gd_p^3 \rho_f^2/\mu_f^2 = Ar\ De/(1 - De) = Ar\ \rho_f/(\rho_s - \rho_f)$
H	bed height, m
n	Richardson–Zaki exponent defined by Eq. (*1*)
Re	Reynolds number, $Re = Ud_p\rho_f/\mu_f$
Re_{mb}	Reynolds number at minimum bubbling condition, $Re_{mb} = U_{mb}d_p\rho_f/\mu_f$
Re_{mf}	Reynolds number at minimum fluidization condition, $Re_{mf} = U_{mf}d_p\rho_f/\mu_f$
Re_t	Reynolds number at terminal velocity of falling particle, $Re_t = U_t d_p \rho_f/\mu_f$
U	superficial velocity of fluid, m s^{-1}
U_e	dynamic (elastic) wave velocity defined by Eq. (*5*), m s^{-1}
U_{mb}	velocity of fluid at minimum bubbling condition, m s^{-1}
U_{mf}	velocity of fluid at minimum fluidization condition, m s^{-1}
U_t	terminal, free fall velocity of particle, m s^{-1}
U_e	continuity (voidage propagation) wave velocity defined by Eq. (*6*), m s^{-1}
W	mass of particles, kg
ε	voidage, void fraction (porosity)
ε_{mb}	voidage at minimum bubbling condition
ε_{mf}	voidage at minimum fluidization condition
μ_f	fluid viscosity, kg m^{-1} s^{-1}
ρ_f	fluid density, kg m^{-3}
ρ_s	solid density, kg m^{-3}

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